Full Bayesian models to handle missing values in cost-effectiveness analysis from individual level data

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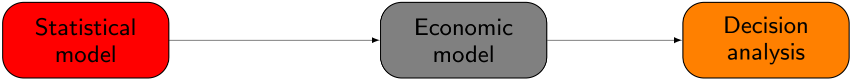
ICER $= \mathsf{E}[\Delta_c]/\mathsf{E}[\Delta_e]$

EIB $= k\mathsf{E}[\Delta_e] - \mathsf{E}[\Delta_c]$

CEAC $= \Pr(k\Delta_e - \Delta_c > 0)$



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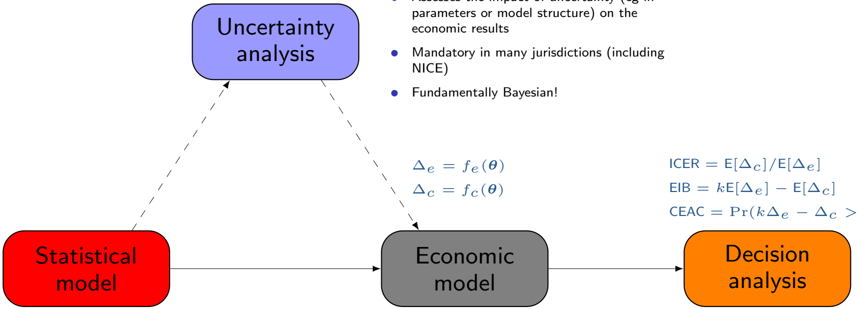

Uncertainty analysis

- Assesses the impact of uncertainty (eg in parameters or model structure) on the economic results
- Mandatory in many jurisdictions (including NICE)
- Fundamentally Bayesian!

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Statistical model — **individual level data**

- The available data usually look something like this:

		Demographics			HRQL data				Resource use data			
ID	**Trt**	Sex	Age	...	u_0	u_1	...	u_J	c_0	c_1	...	c_J
1	1	M	23	...	0.32	0.66	...	0.44	103	241	...	80
2	1	M	21	...	0.12	0.16	...	0.38	1 204	1 808	...	877
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and the **typical** analysis is based on the following steps:

1. Compute individual QALYs and total costs as

$$e_i = \sum_{j=1}^{J} \left(u_{ij} + u_{ij-1}\right) \frac{\delta_j}{2} \quad \text{and} \quad c_i = \sum_{j=0}^{J} c_{ij}, \qquad \left[\text{with: } \delta_j = \frac{\text{Time}_j - \text{Time}_{j-1}}{\text{Unit of time}}\right]$$

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2. (Often implicitly) assume normality and linearity and model **independently** individual QALYs and total costs by controlling for baseline values

$$\begin{aligned} e_i &= \alpha_{e0} + \alpha_{e1} u_{0i} + \alpha_{e2}\text{Trt}_i + \varepsilon_{ei}\,[+\ldots], \qquad \varepsilon_{ei} \sim \text{Normal}(0, \sigma_e) \\ c_i &= \alpha_{c0} + \alpha_{c1} c_{0i} + \alpha_{c2}\text{Trt}_i + \varepsilon_{ci}\,[+\ldots], \qquad \varepsilon_{ci} \sim \text{Normal}(0, \sigma_c) \end{aligned}$$

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3. Estimate population average cost and effectiveness differentials and use bootstrap to quantify uncertainty

What's wrong with this?

- Potential correlation between costs & clinical benefits
  - Strong positive correlation — effective treatments are innovative and result from intensive and lengthy research $\Rightarrow$ are associated with higher unit costs
  - Negative correlation — more effective treatments may reduce total care pathway costs e.g. by reducing hospitalisations, side effects, etc.
  - **NB**: In any case, the economic evaluation is based on both!

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- Joint/marginal normality not realistic
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- ... and of course **Partially Observed** data
  - Can have item and/or unit non-response
  - Missingness may occur in either or both benefits/costs
  - The missingness mechanisms may also be correlated
  - Focus in decision-making — not inference!

To be or not to be (Bayesians)?...

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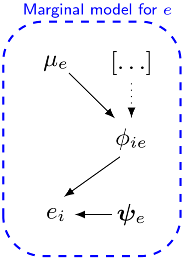
- In general, can represent a joint distribution as a **conditional regression**

$$p(e,c) = p(e)p(c \mid e) = p(c)p(e \mid c)$$



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$g(\phi_{ei}) = \mu_e \, [+ \ldots]$

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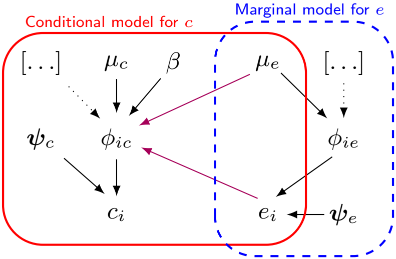

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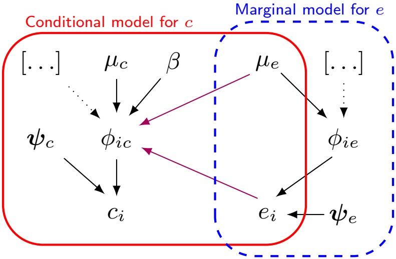

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- For example:

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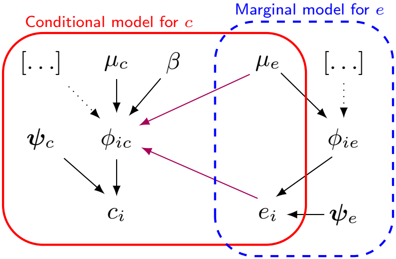

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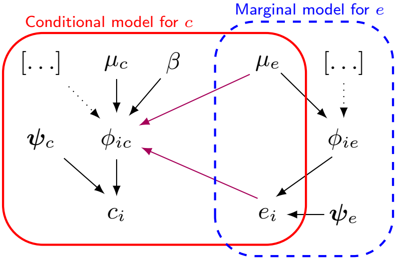

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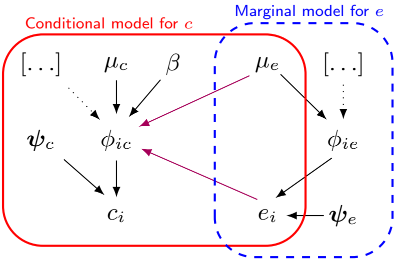

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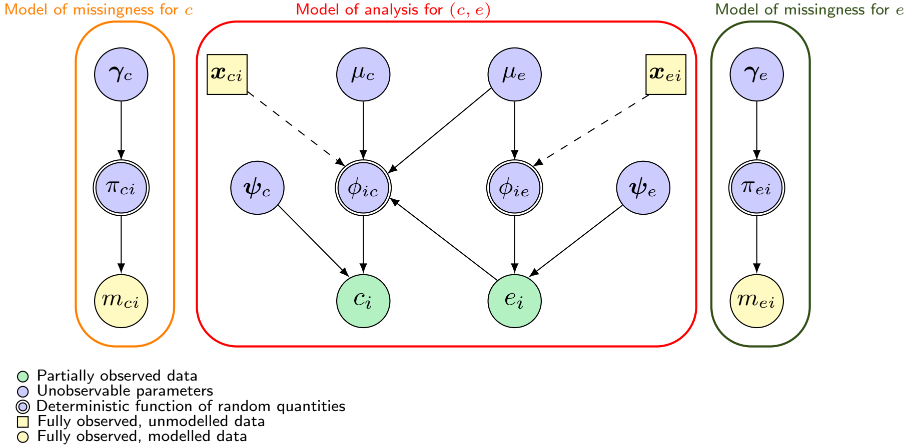
- Combining "modules" and fully characterising uncertainty about deterministic functions of random quantities is relatively straightforward using MCMC
- Prior information can help stabilise inference (especially with sparse data!), eg
  - Cancer patients are unlikely to survive as long as the general population
  - ORs are unlikely to be greater than ± 5

MCAR (e, c)



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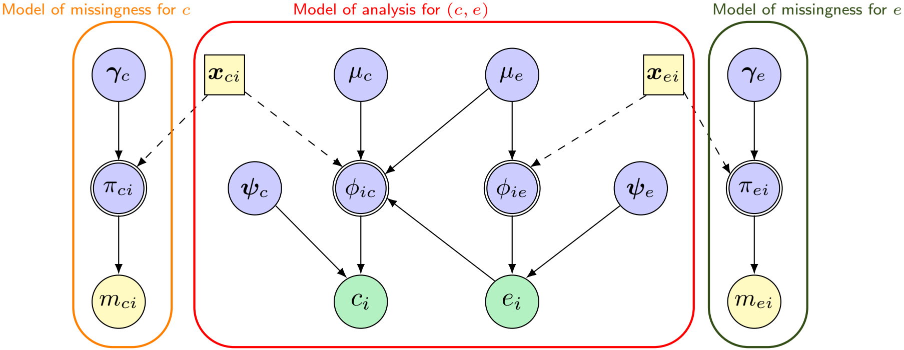

- Partially observed data
- Unobservable parameters
- Deterministic function of random quantities
- Fully observed, unmodelled data
- Fully observed, modelled data

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Dealing with missing data — selection models

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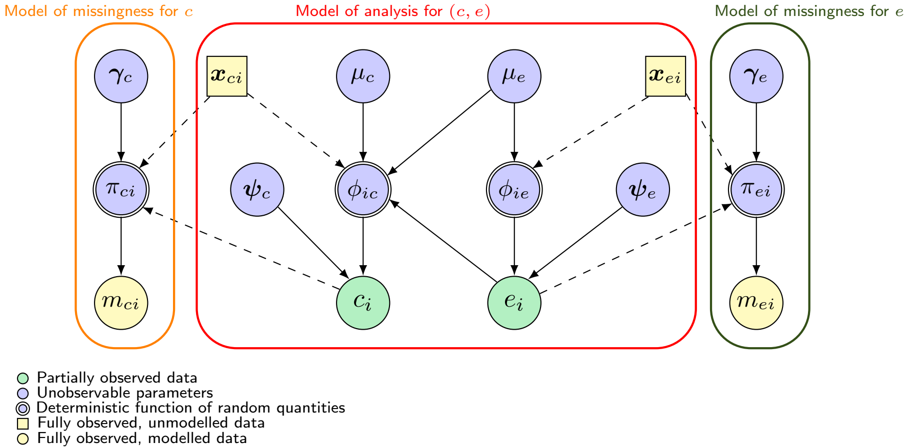

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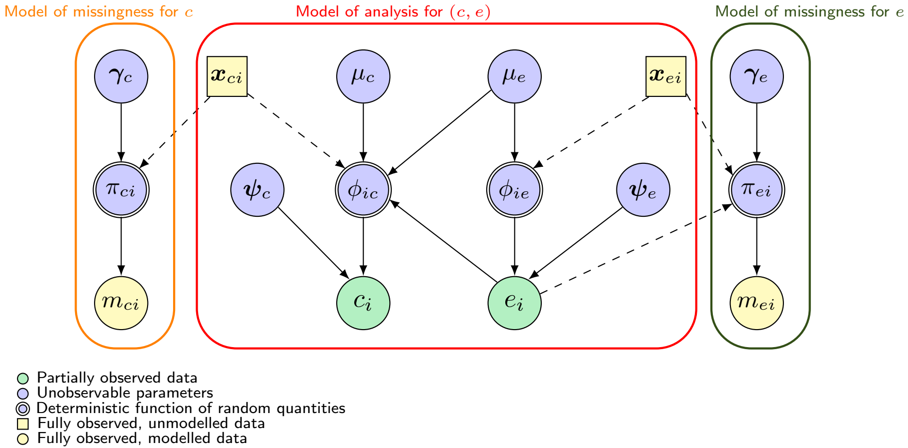

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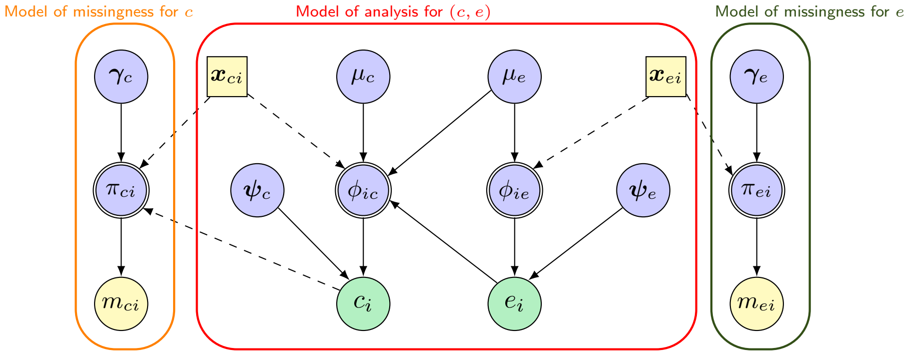

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Motivating example: MenSS trial

- The MenSS pilot RCT evaluates the cost-effectiveness of a new digital intervention to reduce the incidence of STI in young men with respect to the SOC
  - QALYs calculated from utilities (EQ-5D 3L)
  - Total costs calculated from different components (no baseline)

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Time	Type of outcome	observed (%)	observed (%)
		Control (n_1=75)	Intervention (n_2=84)
Baseline	utilities	72 (96%)	72 (86%)
3 months	utilities and costs	34 (45%)	23 (27%)
6 months	utilities and costs	35 (47%)	23 (27%)
12 months	utilities and costs	43 (57%)	36 (43%)
Complete cases	utilities and costs	27 (44%)	19 (23%)

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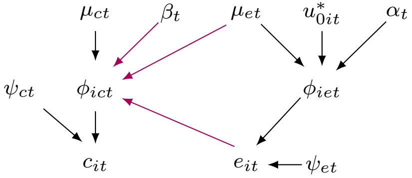
   - Simpler and closer to "standard" frequentist model
   - Account for correlation between QALYs and costs

Conditional model for $c \mid e$

$$c_{it} \mid e_{it} \sim \text{Normal}(\phi_{cit}, \psi_{ct})$$
$$\phi_{cit} = \mu_{ct} + \beta_t(e_{it} - \mu_{et})$$



Marginal model for e

$$e_{it} \sim \text{Normal}(\phi_{eit}, \psi_{et})$$
$$\phi_{eit} = \mu_{et} + \alpha_t(u_{0it} - \bar{u}_{0t})$$
$$= \mu_{et} + \alpha_t u^*_{0it}$$

Modelling

1. **Bivariate Normal**
   - Simpler and closer to "standard" frequentist model
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2. **Beta-Gamma**
   - Account for correlation between outcomes
   - Model the relevant ranges: QALYs $\in (0,1)$ and costs $\in (0,\infty)$
   - **But**: needs to rescale observed data $e^*_{it} = (e_{it} - \epsilon)$ to avoid spikes at 1

Conditional model for $c \mid e^*$

$$c_{it} \mid e^*_{it} \sim \text{Gamma}(\psi_{ct}\phi_{cit}, \psi_{ct})$$
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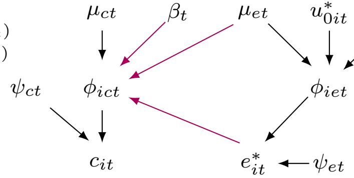

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3. **Hurdle model**
   - Model e_{it} as a **mixture** to account for correlation between outcomes, model the relevant ranges and account for structural values
   - May expand to account for partially observed baseline utility u_{0it}

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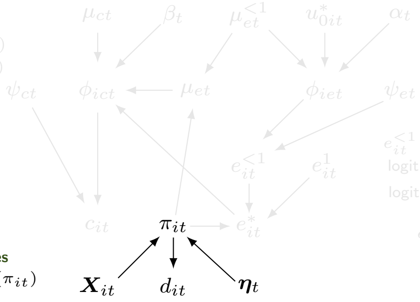

Mixture model for e

$e^1_{it} := 1$

$e^{<1}_{it} \sim \text{Beta}(\phi_{eit}\psi_{et}, (1-\phi_{eit})\psi_{et})$

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Model for the structural ones

$d_{it} := \mathbb{I}(e_{it} = 1) \sim \text{Bernoulli}(\pi_{it})$

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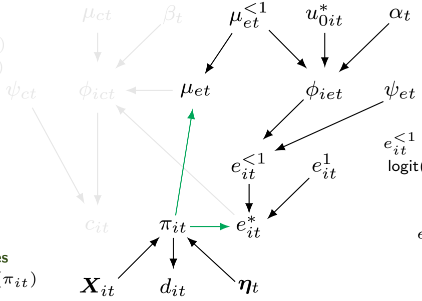
Modelling

1. **Bivariate Normal**
   - Simpler and closer to "standard" frequentist model
   - Account for correlation between QALYs and costs
2. **Beta-Gamma**
   - Account for correlation between outcomes
   - Model the relevant ranges: QALYs $\in (0,1)$ and costs $\in (0,\infty)$
   - **But**: needs to rescale observed data $e^*_{it} = (e_{it} - \epsilon)$ to avoid spikes at 1
3. **Hurdle model**
   - Model e_{it} as a **mixture** to account for correlation between outcomes, model the relevant ranges and account for structural values
   - May expand to account for partially observed baseline utility u_{0it}

Conditional model for $c \mid e^*$

$$c_{it} \mid e^*_{it} \sim \text{Gamma}(\psi_{ct}\phi_{cit}, \psi_{ct})$$
$$\log(\phi_{cit}) = \mu_{ct} + \beta_t(e^*_{it} - \mu_{et})$$



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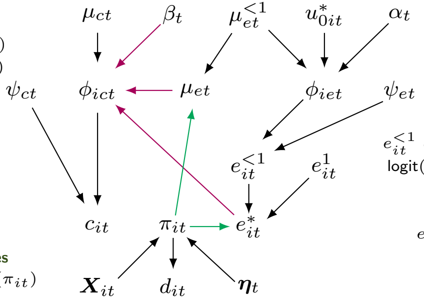

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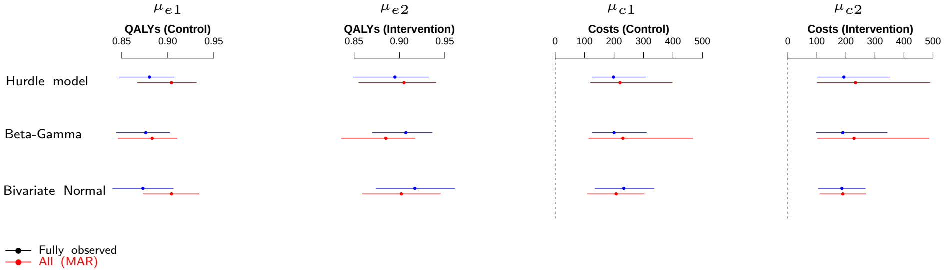

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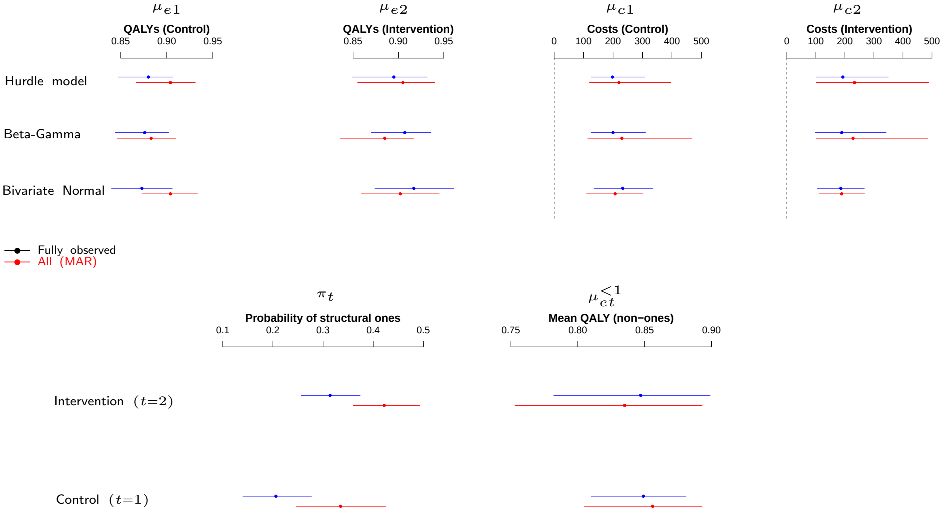
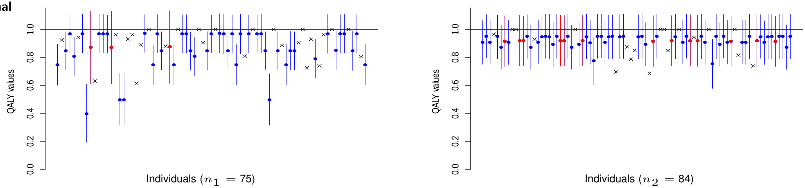


Bayesian multiple imputation (under MAR)

Bivariate Normal

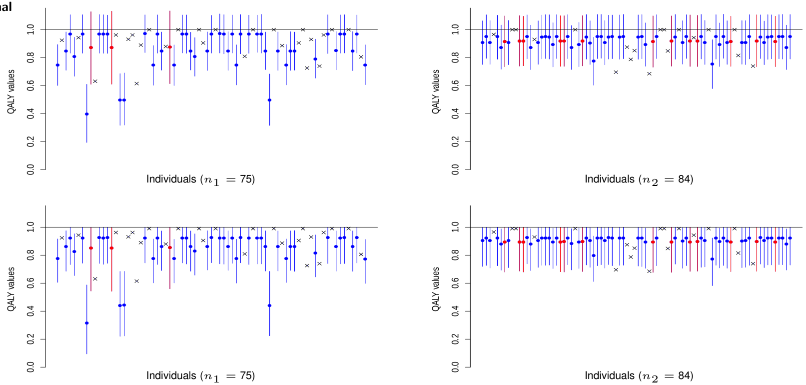


Imputed, observed baseline
Imputed, missing baseline
× Observed

Bayesian multiple imputation (under MAR)

Bivariate Normal

Beta-Gamma



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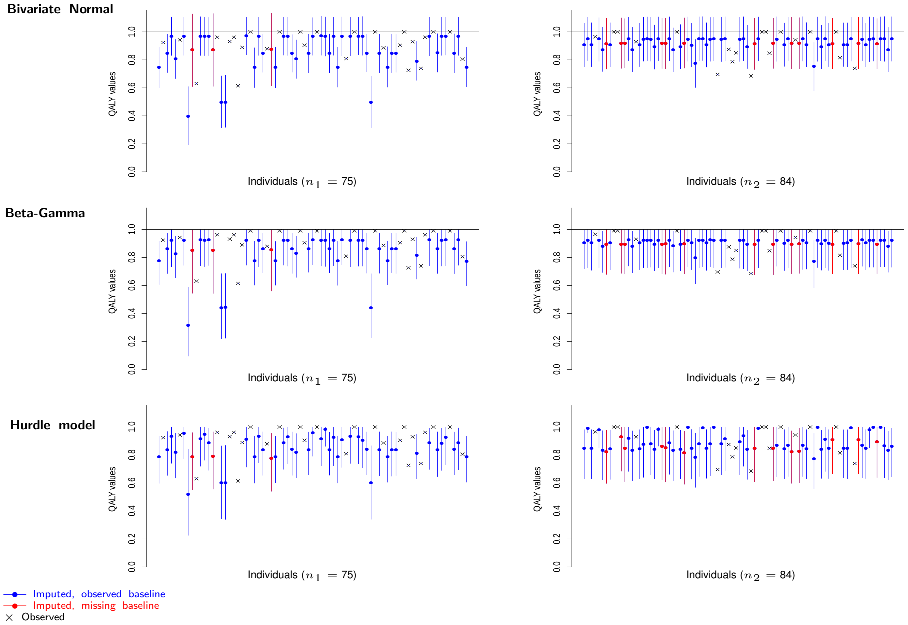

MNAR

- We observe $n_{01} = 13$ and $n_{02} = 22$ individuals with $u_{0it} = 1$ and $u_{jit} = \texttt{NA}$, for $j > 1$
- For those individuals, we cannot compute directly the structural one indicator d_{it} and so need to make assumptions/model this
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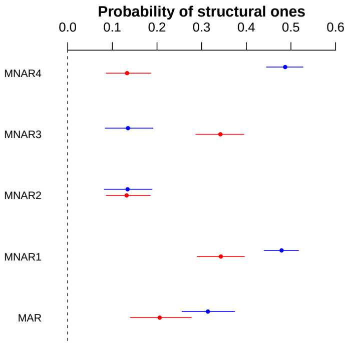
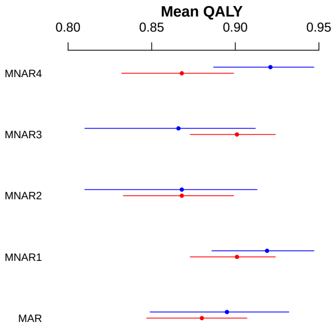
MNAR4. Set $d_{it} = 0$ for the $n_{01} = 13$ individuals with $u_{0i1} = 1$ and $d_{it} = 1$ for the $n_{02} = 22$ individuals with $u_{0i2} = 1$

Results — MNAR

π_t

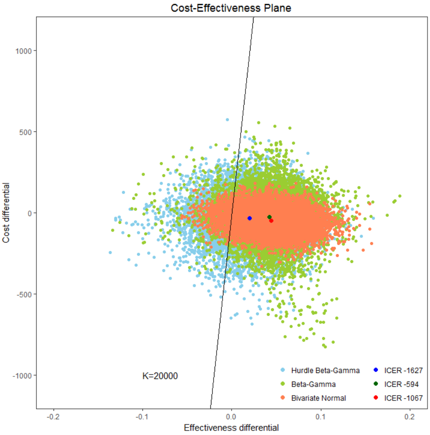


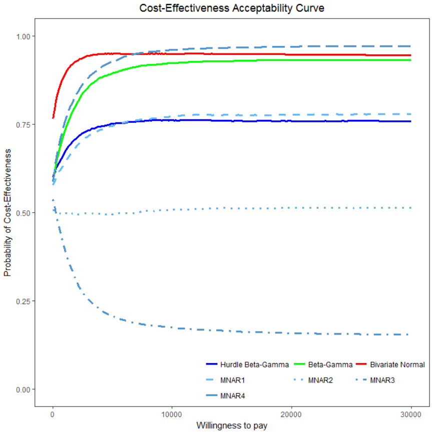
μ_{et}



Control $(t = 1)$
Intervention$(t = 2)$

Cost-effectiveness analysis





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- Particularly helpful in cost-effectiveness analysis, to account for
  - Asymmetrical distributions for the main outcomes
  - Correlation between costs & benefits
  - Structural values (eg spikes at 1 for utilities or spikes at 0 for costs)
- Need specialised software + coding skills
  - `R` package **`missingHE`** under development to implement a set of general models
  - Preliminary work available at `https://github.com/giabaio/missingHE`
  - Eventually, will be able to combine with existing packages (eg **`BCEA`**: `http://www.statistica.it/gianluca/BCEA`; `https://github.com/giabaio/BCEA`) to perform the whole economic analysis

Thank you!